Grade 11/12 Math Circles
February 28, March 6, 2024
Population Modeling - Problem Set

1. Suppose a population of lake trout is growing according to the logistic equation

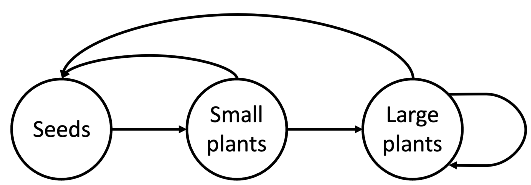
$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right).$$

   (a) What is the maximum possible growth rate for the population? When is it reached? (Write the answer in terms of r and/or K)

   (b) After some investigation on K, a biologist decides to maximize his fishing yield by maintaining a population of lake trout at 1000 individuals. What is the value of K?

   (c) After some more investigation on r, the biologist estimates that $r = 0.01$ individuals/(day$\cdot$ individual). Now if 1200 additional lake trouts are added to the population, what will the instantaneous population growth rate be?

2. Suppose the life history of a plant is shown below:



We initially recorded 100 seeds, 250 small plants, and 50 large plants. After 1 year, we found that 20% of the seeds germinated and grew into small plants, 50% of small plants grew into large plants, and 80% of large plants survived. We also counted that each small plant can produce 5 seeds, while each large plant can produce 20 seeds on average.

   (a) Write down the life table of the plant in a matrix A and the initial population matrix N_0.

   (b) Calculate the number of seeds, small plants, and large plants after 1 year.

   (c) Calculate the number of seeds, small plants, and large plants after 2 years.

   (d) Use an online calculating tool to calculate $A^n N_0$ for $n = 3, 4, 5, 6, 7, 8$. Then plot the natural logarithm (ln) number of individuals in each stage class versus time. What did you find?